Grade 8 Mathematics, Quarter 4, Unit 4.1

Analyzing and Using Systems of Linear Equations with Word Problems

Overview

Number of instructional days: 15 (1 day = 45–60 minutes)

Content to be learned

- Understand that the point of intersection of two linear equations is the solution to the system of equations.
- Determine the solution to a system of equations both graphically and algebraically.
- Use a system of equations to solve real-world problems.
- Know that the solution of a system is the point of intersection of the graphs of the two equations in the system, and the coordinates of the point satisfy both equations.
- Recognize that a system of bivariant equations must have two equations.
- Understand the three cases found in systems of equations, (one solution, no solution, infinite solutions).

Mathematical practices to be integrated

Reason abstractly and quantitatively.

- Make sense of quantities and their relationships in problem situations involving more than one set of conditions.
- Understand that quantitative reasoning entails habits of creating a coherent representation of the problem at hand, considering the units involved.

Model with mathematics.

- Apply the mathematics known to solve problems arising in everyday life, society, and the workplace through the making of a system to represent the problem situation.
- Identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas.

Attend to precision.

- Communicate precisely to others when explaining what the solution of a system of equations means.
- Calculate accurately and efficiently and express numerical answers with a degree of precision appropriate for the problem context.

Look for and make use of structure.

- Look closely to discern a pattern or structure.
- Understand complicated things, such as some algebraic expressions, as single objects or as being composed of several objects.

Essential questions

- How do you determine the solution to a system of linear equations graphically?
- How do you determine the solution to a system of linear equations algebraically?
- What is an explanation of how to construct a system of linear equations with infinitely many solutions?
- How do you construct a system of linear equations with exactly one solution?
- What would cause a system of linear equations to have no solution?
- How would you prove that a given coordinate (x, y) is a solution to a system of linear equations?

Written Curriculum

Common Core State Standards for Mathematical Content

Expressions and Equations	8.EE

Analyze and solve linear equations and pairs of simultaneous linear equations.

8.EE.8 Analyze and solve pairs of simultaneous linear equations.

a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.*

c. Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*

Common Core Standards for Mathematical Practice

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Clarifying the Standards

Prior Learning

In grade 2, students began representing and interpreting data and continued through grade 7. In grades 4 and 5, students generated and analyzed patterns and relationships. In grade 5, students wrote and interpreted numerical expressions. In grade 6, students wrote, read, and evaluated expressions in which letters stand for numbers. In grade 7, students solved multi-step, real-life, and mathematical problems using positive and negative rational numbers in any form.

Current Learning

Students solve a system of linear equations both graphically and algebraically.

Future Learning

Students will solve a simple system consisting of a linear equation and a nonlinear equation (such as exponential and quadratic) in two variables, algebraically and graphically.

Additional Findings

When students work toward meeting this standard, they build on what they know about two-variable linear equations, and they enlarge the varieties of real-world and mathematical problems they can solve. Nonlinear functions do not need to be taught, but there should be sufficient treatment of nonlinear functions to avoid giving students the misleading impression that all functional relationships are linear.

Grade 8 Mathematics, Quarter 4, Unit 4.2

Using Scatterplots to Interpret Linear Associations

Overview

Number of instructional days: 15 (1 day = 45–60 minutes)

Content to be learned

- Understand that linear equations can represent a relationship between two variables on a scatterplot.
- Recognize that a linear model is one way of determining a pattern in a scatterplot.
- Use a linear equation to interpret what the slope and y-intercept represent.
- Predict a possible correlation between two variables that are represented in the scatterplot.
- Use a linear equation to solve unknowns in a given situation.

Mathematical practices to be integrated

Model with mathematics.

- Identify important quantities in practical situations and map their relationships using diagrams, two-way tables, graphs, flowcharts, and formulas.
- Apply the mathematics known to solve problems arising in everyday life, society, and the workplace through the making of a system to represent the problem situation.

Use appropriate tools strategically.

- Analyze graphs of functions and solutions generated using a graphing calculator.
- Consider the available tools when solving a mathematical problem.

Essential questions

- What can data clustering reveal on a scatterplot?
- How can the estimate of the line of best fit be used to make predictions about the set of data?
- How can you use a scatterplot to draw informal inference between two variables?
- When estimating the line of best fit, how should the line be positioned?
- When is a scatterplot appropriate for displaying data?
- What types of information can be gathered from a two-way table?
- What kind of data is displayed in a two-way table?

Written Curriculum

Common Core State Standards for Mathematical Content

Statistics and Probability	8.SP

Investigate patterns of association in bivariate data.

8.SP.2 Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.

8.SP.3 Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.*

8.SP.4 Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*

Common Core Standards for Mathematical Practice

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

Clarifying the Standards

Prior Learning

In grades 1–5, students represented and interpreted data. In grade 4, students generated and analyzed patterns. In grade 5, students, wrote and interpreted numerical expressions, and they generated and analyzed patterns and relationships. Also in grade 5, students graphed points on the coordinate plane to solve real-world and mathematical problems. In grade 6, students wrote, read, and evaluated expressions in which letters stand for numbers. They also represented and analyzed quantitative relationships between independent and dependent variables. In grade 7, students solved multi-step, real-life, and mathematical problems, including positive and negative rational numbers in any form.

Current Learning

Students make scatterplots from two variables and write a linear equation that models the information. They also interpret the meaning of slope and y-intercept. Students construct and interpret a two-way table.

Future Learning

In high school, students will represent and model with vector quantities and create equations that describe numbers and relationships. They will also build a function that models a relationship between two quantities. In statistics, students will summarize, represent, and interpret data on two categorical variables as well as interpret linear models.

Additional Findings

When students work toward meeting this standard, they build on what they know about two-variable linear equations, and they expand the varieties of real-world and mathematical problems they can solve. Nonlinear functions do not need to be taught, but there should be sufficient treatment of nonlinear functions to avoid giving students the misleading impression that all functional relationships are linear.